Trinity College

WA Exams Practice Paper C, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section One: Calculator-free

SOLUTIONS

Student Number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

(a) If $P=\begin{bmatrix}0 & 3\\-2 & 1\end{bmatrix}$ and $Q=\begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}$ determine

(i) $2PQ$. (2 marks)

$$2\begin{bmatrix}0 & 3\\-2 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}=2\begin{bmatrix}6 & -9\\0 & -1\end{bmatrix}=\begin{bmatrix}12 & -18\\0 & -2\end{bmatrix}$$

(ii) Q^{-1}. (2 marks)

$$\begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}^{-1}=\frac{1}{-3-(-2)}\begin{bmatrix}-3 & 1\\-2 & 1\end{bmatrix}=\begin{bmatrix}3 & -1\\2 & -1\end{bmatrix}$$

(b) Show use of matrix methods to solve the simultaneous equations $2x+y=8$ and $-x+y=2$. (3 marks)

$$\begin{bmatrix}2 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\2\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & -1\\1 & 2\end{bmatrix}\begin{bmatrix}8\\2\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\4\end{bmatrix}$$

Question 2 **(8 marks)**

Let the vector $\mathbf{a} = 4\mathbf{i} + (k-1)\mathbf{j}$ and the vector $\mathbf{b} = (k+3)\mathbf{i} + 3\mathbf{j}$, where k is a constant.

(a) Determine, in exact form, a vector $\mathbf{c}$ that has the same magnitude as $\mathbf{a}$ and is parallel to $\mathbf{b}$ when $k = -2$. (3 marks)

$\mathbf{a} = 4\mathbf{i} - 3\mathbf{j}$

$|\mathbf{a}| = 5$

$\mathbf{b} = \mathbf{i} + 3\mathbf{j}$

$|\mathbf{b}| = \sqrt{10}$

$\mathbf{c} = \frac{5}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j})$

$= \frac{\sqrt{10}}{2}(\mathbf{i} + 3\mathbf{j})$ or $-\frac{\sqrt{10}}{2}(\mathbf{i} + 3\mathbf{j})$

(b) Determine the value(s) of k if $\mathbf{a}$ and $\mathbf{b}$ are parallel. (3 marks)

$\mathbf{a} = \lambda \mathbf{b} \Rightarrow \frac{4}{k+3} = \frac{k-1}{3}$

$k = -5,\ 3$

(c) Can $\mathbf{a} + \mathbf{b} = \mathbf{0}$? Justify your answer. (2 marks)

No.

By equating $\mathbf{i}$ and $\mathbf{j}$ coefficients to zero we require

$\mathbf{i}: 4 + k + 3 = 0 \Rightarrow k = -7$

$\mathbf{j}: k - 1 + 3 = 0 \Rightarrow k = -2$

Question 3 **(8 marks)**

(a) Express $\sin\theta + \sqrt{3}\cos\theta$ in the form $a\sin(\theta + b)$. (2 marks)

$$a = \sqrt{1+3} = 2$$
$$b = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$$
$$2\sin\left(\theta + \frac{\pi}{3}\right)$$

(b) Express $\cos(15°) - \cos(105°)$ as an exact value. (3 marks)

$$\cos(15°) - \cos(105°) = -2\sin\left(\frac{15+105}{2}\right)\sin\left(\frac{15-105}{2}\right)$$
$$= -2\sin(60)\sin(-45)$$
$$= 2\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}}{2}$$

(c) Solve $\sin\theta = \sin(2\theta)$ for $0 \le \theta \le 360°$. (3 marks)

$$\sin 2\theta - \sin\theta = 0$$
$$2\cos\frac{3\theta}{2}\sin\frac{\theta}{2} = 0$$
$$\sin\frac{\theta}{2} = 0 \qquad \cos\frac{3\theta}{2} = 0$$
$$\theta = 0, 360 \qquad \theta = 60, 180, 300$$
$$\theta = 0, 60, 180, 300, 360$$

Question 4 **(8 marks)**

(a) Determine what fraction of all possible arrangements of the letters of the word SELECTIONS begin with the letters EE. (3 marks)

$$\left(\frac{8!}{2!}\right) \div \left(\frac{10!}{2!2!}\right) = \frac{8!}{2!} \times \frac{2!2!}{10!}$$
$$= \frac{2}{10 \times 9}$$
$$= \frac{1}{45}$$

(b) A team of four is to be chosen from six girls and five boys. In how many ways can this be done, if there are to be more girls than boys. (3 marks)

$$^6C_4 = \frac{6!}{2!4!} = \frac{6 \times 5}{2 \times 1} = 15$$

$$^6C_3 \times {}^5C_1 = \frac{6!}{3!3!} \times \frac{5!}{1!4!}$$
$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 5$$
$$= 100$$

$$100 + 15 = 115$$

(c) A student had five different text-books on their table. Given that they took at least one but not all of them, how many possible combinations of text-books could they have made to take to school? (2 marks)

$$\sum_{r=1}^{4} {}^5C_r = 2^5 - 1 - 1$$
$$= 30$$

Question 5 **(6 marks)**

The three points A, B and C have position vectors $\mathbf{a} = 13\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = -35\mathbf{i} + 17\mathbf{j}$ and $\mathbf{c} = \mathbf{i} + m\mathbf{j}$ respectively. The point C divides AB internally in the ratio $1{:}n$. Find the values of m and n.

$$(1+n)\,\mathbf{AC} = \mathbf{AB}$$

$$(1+n)\left(\begin{bmatrix} 1 \\ m \end{bmatrix} - \begin{bmatrix} 13 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} -35 \\ 17 \end{bmatrix} - \begin{bmatrix} 13 \\ -3 \end{bmatrix}$$

$$(1+n)\begin{bmatrix} -12 \\ m+3 \end{bmatrix} = \begin{bmatrix} -48 \\ 20 \end{bmatrix}$$

$$(1+n)(-12) = -48$$
$$\therefore n = 3$$

$$4(m+3) = 20$$
$$\therefore m = 2$$

Question 6 **(7 marks)**

(a) One of the complex roots of a real quadratic equation is $2+i$. Determine the real quadratic equation in the form $ax^2+bx+c=0$. (2 marks)

$$(x-2-i)(x-2+i)=0$$
$$x^2-4x+4-i^2=0$$
$$x^2-4x+5=0$$

(b) If $z=3-i$, express $\frac{z}{\overline{z}}$ in simplified form. (2 marks)

$$\frac{3-i}{3+i}\times\frac{3-i}{3-i}=\frac{8-6i}{10}$$
$$=\tfrac{4}{5}-\tfrac{3}{5}i$$

(c) Determine the complex number z, if $2\overline{z}-iz+4+i=0$. (3 marks)

$$z=x+iy$$

$$2(x-iy)-i(x+iy)+4+i=0$$
$$2x-2iy-ix+y+4+i=0$$

Real parts $2x+y+4=0$

Imaginary parts $-2y-x+1=0 \Rightarrow x=1-2y$

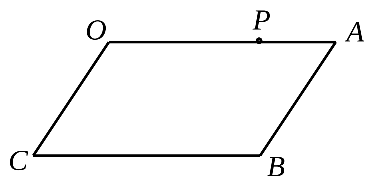
$$2(1-2y)+y=-4$$
$$-3y=-6$$
$$y=2$$
$$x=-3$$

$$z=-3+2i$$

Question 7 **(8 marks)**

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

P is the point on side OA such that $OP : PA = 3:1$.



(a) Express in terms of $\mathbf{a}$ and $\mathbf{c}$:

(i) $\overrightarrow{OB}$. (1 mark)

$OB = \mathbf{a} + \mathbf{c}$

(ii) $\overrightarrow{OP}$. (1 mark)

$OP = \frac{3}{4}\mathbf{a}$

(iii) $\overrightarrow{CP}$. (1 mark)

$CP = \frac{3}{4}\mathbf{a} - \mathbf{c}$

N is the point on CP such that O, N and B are collinear.

(b) If $\overrightarrow{ON} = k \cdot \overrightarrow{OB}$ and $\overrightarrow{CN} = h \cdot \overrightarrow{CP}$, use the fact that $\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$ to determine the values of h and k. (5 marks)

$$ON = OC + CN$$
$$k \cdot OB = \mathbf{c} + h \cdot CP$$
$$k(\mathbf{a} + \mathbf{c}) = \mathbf{c} + h(\tfrac{3}{4}\mathbf{a} - \mathbf{c})$$

Consider $\mathbf{a}$ coefficients: $k = \frac{3}{4}h$

Consider $\mathbf{c}$ coefficients: $k = 1 - h$

Solving simultaneously gives: $k = \frac{3}{7}$, $h = \frac{4}{7}$

End of questions

Additional working space

Question number: _________

Additional working space

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